

CS 405/505 ASSIGNMENT #8

Due Thursday, November 1, 2007 ¹

1. Roman numerals use the following symbols:

I (one)
V (five)
X (ten)
L (fifty)
C (one hundred)
D (five hundred)
M (one thousand)

These are usually written in linear order (e.g., I, II, III) with smaller numerals always following larger numerals except when they are to be subtracted (e.g., IV means 5 - 1). Subtractions are used instead of writing four consecutive occurrences of the same symbol (i.e., IV should be used instead of IIII, MCM instead of MDCCCC, etc.). Only symbols which are powers of 10 may be used in subtractions (i.e., I, X, C, and M). Furthermore, subtraction rules require that a symbol representing 10^x may not precede any symbol larger than 10^{x+1} (e.g., IC is not permitted to represent 99 but XC is 90). Write a Prolog predicate `romanToDecimal(RomanNumeral, DecimalNumber)` which takes a Roman numeral as a list of digits and returns the decimal equivalent if the Roman numeral is well formed and fails otherwise. Test your predicate as follows:

- (a) `romanToDecimal([c, d, v], N) ⇒ N = 405`
- (b) `romanToDecimal([m, m, v, i, i], N) ⇒ N = 2007`
- (c) `romanToDecimal([m, m, m, c, m, x, c, i, x], N) ⇒ N = 3999`
- (d) `romanToDecimal([m, i, m], N) ⇒ no`

2. DNA is a two-stranded molecule. Each strand is a polynucleotide composed of A (adenosine), T (thymidine), C (cytidine), and G (guanosine) residues. The two strands of DNA run antiparallel (i.e., at each nucleotide residue along the double-stranded DNA molecule, the nucleotides are complementary). That is, A forms two hydrogen-bonds with T; C forms three hydrogen bonds with G. In most cases the two-stranded, antiparallel, complementary DNA molecule folds to form a helical structure, which resembles a spiral staircase. This is the reason why DNA has been referred to as the “Double Helix.” An example of two complementary strands of DNA would be:

```
ATGGAATTCTCGCTC
TACCTTAAGAGCGAG
```

This genetic sequencing problem is to find complementary matching strands of DNA. Write a Prolog predicate `matchDNA(Strand1, Strand2, Match)` that takes two strands as lists of nucleotides, and returns in `Match`: 1) the beginning part of the second strand which does not match, 2) the part of the second strand which the first string matches, and 3) the remaining part of the second strand which does not match. Note that if there is no match, the second and third components will be null. If there are multiple matching strings, returning only the first match is sufficient. Test your predicate as follows:

¹This is the absolute deadline for turning in this assignment. No late assignments will be accepted.

- (a) $\text{matchDNA}([a, t, g, c], [a, t, g, c, a, t, a, c, g, a], M) \Rightarrow$
 $M = [[a, t, g, c, a], [t, a, c, g], [a]]$
- (b) $\text{matchDNA}([a], [a, t, g, c, a, t], M) \Rightarrow M = [[a], [t], [g, c, a, t]]$
- (c) $\text{matchDNA}([c, g], [a, t, g, c], M) \Rightarrow M = [[a, t], [g, c], []]$
- (d) $\text{matchDNA}([g, a, a, t, t, c], [t, g, g, a, a, t, t, c, t], M) \Rightarrow$
 $M = [[t, g, g, a, a, t, t, c, t], [] []]$

3. Consider the Core program below:

```

a := m; b := n;
k := 1;
while (b > 0) loop
  b := b - 1;
  k := k * a;
end loop;

```

Using the axiomatic semantics of Core, show that the assertion

$$\{m > 0 \ \& \ n \geq 0\} \ S \ \{k = m^n\}$$

is correct.

Hint: Use the loop invariant $b \geq 0 \ \& \ k \times a^b = m^n$. The consequence rules may be used to force the other rules into a form consistent with this loop invariant. (You should analyze the program to understand why this loop invariant was chosen.)