

INTERPRETATION USING DENOTATIONAL SEMANTICS

$$\begin{aligned}
 & \underline{M[\text{input } x; x := x + 1; \text{output } x] \langle 4, 1, 7 \rangle} \\
 & (\underline{S[\text{input } x; x := x + 1; \text{output } x] \langle \lambda V.\perp, \langle 4, 1, 7 \rangle, nil \rangle}) \downarrow 3 \\
 & (S[x := x + 1; \text{output } x] (\underline{S[\text{input } x] \langle \lambda V.\perp, \langle 4, 1, 7 \rangle, nil \rangle})) \downarrow 3 \\
 & (S[x := x + 1; \text{output } x] \\
 & \quad (\underline{\text{if } \langle 4, 1, 7 \rangle = nil \text{ then } \top \text{ else } \langle \lambda V.\perp[hd(\langle 4, 1, 7 \rangle)/x], \langle 1, 7 \rangle, nil \rangle})) \downarrow 3 \\
 & (S[x := x + 1; \text{output } x] \langle \lambda V.\perp[hd(\langle 4, 1, 7 \rangle)/x], \langle 1, 7 \rangle, nil \rangle) \downarrow 3 \\
 & (\underline{S[x := x + 1; \text{output } x] \langle \lambda V.\perp[4/x], \langle 1, 7 \rangle, nil \rangle}) \downarrow 3 \\
 & (S[\text{output } x] (\underline{S[x := x + 1] \langle \lambda V.\perp[4/x], \langle 1, 7 \rangle, nil \rangle})) \downarrow 3 \\
 & (S[\text{output } x] (\langle \lambda V.\perp[4/x][E[x + 1] (\lambda V.\perp[4/x])/x], \langle 1, 7 \rangle, nil \rangle)) \downarrow 3 \\
 & (S[\text{output } x] (\langle \lambda V.\perp[4/x][(\underline{E[x]} (\lambda V.\perp[4/x]) + E[1] (\lambda V.\perp[4/x]))/x], \langle 1, 7 \rangle, nil \rangle)) \downarrow 3 \\
 & (S[\text{output } x] \\
 & \quad (\langle \lambda V.\perp[4/x][(\text{if } (\lambda V.\perp[4/x]) [x] = \perp \text{ then } \top \\
 & \quad \text{else } (\lambda V.\perp[4/x]) [x] + E[1] (\lambda V.\perp[4/x])/x], \\
 & \quad \langle 1, 7 \rangle, nil \rangle)) \downarrow 3 \\
 & (S[\text{output } x] (\langle \lambda V.\perp[4/x][(\text{if } 4 = \perp \text{ then } \top \text{ else } 4) + E[1] (\lambda V.\perp[4/x])/x], \langle 1, 7 \rangle, nil \rangle)) \downarrow 3 \\
 & (S[\text{output } x] (\langle \lambda V.\perp[4/x][(4 + \underline{E[1]} (\lambda V.\perp[4/x]))/x], \langle 1, 7 \rangle, nil \rangle)) \downarrow 3 \\
 & (S[\text{output } x] (\langle \lambda V.\perp[4/x][(4 + 1)/x], \langle 1, 7 \rangle, nil \rangle)) \downarrow 3 \\
 & (\underline{S[\text{output } x] (\langle \lambda V.\perp[4/x][5/x], \langle 1, 7 \rangle, nil \rangle)}) \downarrow 3 \\
 & (\underline{\text{if } (\lambda V.\perp[4/x][5/x]) [x] = \perp \text{ then } \top \\
 & \quad \text{else } \langle \lambda V.\perp[4/x][5/x], \langle 1, 7 \rangle, \text{append nil (list ((\lambda V.\perp[4/x][5/x]) [x])) \rangle}}) \downarrow 3 \\
 & (\underline{\text{if } (\lambda V.\perp[4/x][5/x]) [x] = \perp \text{ then } \top \\
 & \quad \text{else } \langle \lambda V.\perp[4/x][5/x], \langle 1, 7 \rangle, \text{append nil (list (\lambda V.\perp[4/x][5/x][x])) \rangle}}) \downarrow 3 \\
 & (\underline{\text{if } 5 = \perp \text{ then } \top \text{ else } \langle \lambda V.\perp[4/x][5/x], \langle 1, 7 \rangle, \text{append nil (list 5)} \rangle}}) \downarrow 3 \\
 & (\underline{\langle \lambda V.\perp[4/x][5/x], \langle 1, 7 \rangle, \langle 5 \rangle \rangle}) \downarrow 3 \\
 & \langle 5 \rangle
 \end{aligned}$$