

## CS 401/501 ASSIGNMENT #5

Due Thursday, April 2, 2009

1. Write a Prolog predicate `listFromTo(M, N, List)`, where `M` and `N` are integers and `List` is a list of all integers from `M` to `N`. Test your predicate on the following:

```
listFromTo(2, 5, List) ⇒ List = [2, 3, 4, 5]
listFromTo(-1, -10, List) ⇒ List = []
listFromTo(-10, -1, List) ⇒ List = [-10, -9, ..., -1]
```

2. Write a Prolog predicate `removeMultiples(List1, N, List2)`, where `List1` is a list of integers, `N` is an integer, and `List2` is `List1` with all elements which are divisible by `N` removed. Test your predicate on the following:

```
removeMultiples([2, 3, 4, 5], 2, List) ⇒ List = [3, 5]
removeMultiples([5, 10, 15], 5, List) ⇒ List = []
removeMultiples([2, 3, 5, 7], 9, List) ⇒ List = [2, 3, 5, 7]
```

3. Write a Prolog predicate `sieveOfEratosthenes(N, Primes)`, where `N` is an integer and `Primes` is a list of all prime numbers between 2 and `N`. The predicate should use the Sieve of Eratosthenes to compute the prime numbers as follows:

Begin with a list of integers from 2 to `N`. The first integer, 2, is the first prime. Remove every multiple of this prime. Repeatedly take the next integer in the list as the next prime and remove every multiple of the prime.

Test your predicate on the following:

```
sieveOfEratosthenes(1, Primes) ⇒ Primes = []
sieveOfEratosthenes(5, Primes) ⇒ Primes = [2, 3, 5]
sieveOfEratosthenes(30, Primes) ⇒ Primes = [2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
```

4. Consider the Core program segment `S` below:

```
odd := 1; sum := 1;
while (odd <> 2 * n - 1) loop
  odd := odd + 2;
  sum := sum + odd;
end loop;
```

Using the axiomatic semantics of Core, show that the assertion

$$\{n \geq 1\} S \{\text{sum} = n^2\}$$

is correct.

Hint: Use the loop invariant  $\text{sum} = \left(\frac{\text{odd}+1}{2}\right)^2 \ \& \ \text{odd} \geq 1 \ \& \ \text{odd} \bmod 2 = 1$ . The consequence rules may be used to force the other rules into a form consistent with this loop invariant. (You should analyze the program to understand why this loop invariant was chosen.)