

## DENOTATIONAL DEFINITION OF MINI-LANGUAGE CORE SUBSET †

### Concrete Syntax of Core Subset

$\langle \text{statement sequence} \rangle ::= \langle \text{statement} \rangle ; \{ \langle \text{statement} \rangle ; \}$   
 $\langle \text{statement} \rangle ::= \langle \text{assignment statement} \rangle \mid \langle \text{if statement} \rangle \mid \langle \text{loop statement} \rangle$   
 $\quad \mid \langle \text{input statement} \rangle \mid \langle \text{output statement} \rangle$   
 $\langle \text{assignment statement} \rangle ::= \langle \text{identifier} \rangle := \langle \text{expression} \rangle$   
 $\langle \text{if statement} \rangle ::= \text{if } \langle \text{comparison} \rangle \text{ then } \langle \text{statement sequence} \rangle [\text{else } \langle \text{statement} \rangle] \text{ end if}$   
 $\langle \text{while statement} \rangle ::= \text{while } \langle \text{comparison} \rangle \text{ loop } \langle \text{statement sequence} \rangle \text{ end loop}$   
 $\langle \text{input statement} \rangle ::= \text{input } \langle \text{identifier} \rangle$   
 $\langle \text{output statement} \rangle ::= \text{output } \langle \text{identifier} \rangle$   
 $\langle \text{comparison} \rangle ::= ( \langle \text{factor} \rangle \langle \text{comparison operator} \rangle \langle \text{factor} \rangle )$   
 $\langle \text{expression} \rangle ::= \langle \text{term} \rangle \{ \langle \text{add operator} \rangle \langle \text{term} \rangle \}$   
 $\langle \text{term} \rangle ::= \langle \text{factor} \rangle \{ * \langle \text{factor} \rangle \}$   
 $\langle \text{factor} \rangle ::= ( \langle \text{expression} \rangle ) \mid \langle \text{identifier} \rangle \mid \langle \text{integer} \rangle$   
 $\langle \text{comparison operator} \rangle ::= < \mid \leq \mid = \mid \neq \mid > \mid \geq$   
 $\langle \text{add operator} \rangle ::= + \mid -$

### Syntax Domains

<b>P</b> : Prog	programs	<b>S</b> : Stmt	statements
<b>E</b> : Exp	expressions	<b>C</b> : Comp	comparisons
<b>R</b> : Rel	relations	<b>O</b> : Opr	operations
<b>I</b> : Num	integer constants	<b>V</b> : Var	variables

### Abstract Syntax Rules

$\mathbf{P} ::= \mathbf{S}$   
 $\mathbf{S} ::= \mathbf{S}_1 ; \mathbf{S}_2 \mid \mathbf{V} := \mathbf{E} \mid \text{while } \mathbf{C} \text{ loop } \mathbf{S} \text{ end loop} \mid \text{if } \mathbf{C} \text{ then } \mathbf{S} \text{ end if} \mid \text{if } \mathbf{C} \text{ then } \mathbf{S}_1 \text{ else } \mathbf{S}_2 \text{ end if}$   
 $\quad \mid \text{input } \mathbf{V} \mid \text{output } \mathbf{V}$   
 $\mathbf{C} ::= \mathbf{E}_1 \mathbf{R} \mathbf{E}_2$   
 $\mathbf{E} ::= \mathbf{E}_1 \mathbf{O} \mathbf{E}_2 \mid \mathbf{V} \mid \mathbf{I}$   
 $\mathbf{R} ::= < \mid \leq \mid = \mid \neq \mid > \mid \geq$   
 $\mathbf{O} ::= + \mid - \mid *$

### Semantic Domains

$conf$	: $\text{Cf} = \text{S} \times \text{Fi} \times \text{Fi}$	$file$	: $\text{Fi} = \mathbf{N}^*$
$store$	: $\text{S} = \text{Var} \rightarrow \mathbf{N}$	$t$	: $\text{T} = \{true, false\}^\circ$
$n$	: $\mathbf{N} = \{\dots, -2, -1, 0, 1, 2, \dots\}^\circ$		

† M. Marcotty and H. F. Ledgard, *Programming Language Landscape: Syntax, Semantics, and Implementation*, 2nd ed., SRA, 1986.

## Semantic Functions

$M : \text{Prog} \rightarrow \text{Fi} \rightarrow \text{Fi}$   
 $C : \text{Comp} \rightarrow \text{S} \rightarrow \text{T}$   
 $N : \text{Num} \rightarrow \text{N}$

$S : \text{Stmt} \rightarrow \text{Cf} \rightarrow \text{Cf}$   
 $E : \text{Exp} \rightarrow \text{S} \rightarrow \text{N}$

## Semantic Equations

$$M[[\mathbf{P}]]file_1 = (S[[\mathbf{P}]](\langle \lambda \mathbf{V} . \perp, file_1, nil \rangle)) \downarrow 3$$

$$S[[\mathbf{S}_1 ; \mathbf{S}_2]]conf = S[[\mathbf{S}_2]](S[[\mathbf{S}_1]]conf)$$

$$S[[\mathbf{V} := \mathbf{E}]]conf = \langle store [E [[\mathbf{E}]]store / \mathbf{V}], file_1, file_2 \rangle$$

$$\text{where } store = conf \downarrow 1, file_1 = conf \downarrow 2, file_2 = conf \downarrow 3$$

$$S[[\mathbf{while} \mathbf{C} \text{ loop } \mathbf{S} \text{ end loop}]]conf = \\ \text{if } C[[\mathbf{C}]](conf \downarrow 1) \text{ then} \\ \quad S[[\mathbf{while} \mathbf{C} \text{ loop } \mathbf{S} \text{ end loop}]](S[[\mathbf{S}]]conf) \\ \text{else} \\ \quad conf$$

$$S[[\mathbf{if} \mathbf{C} \text{ then } \mathbf{S} \text{ end if}]]conf = \text{if } C[[\mathbf{C}]](conf \downarrow 1) \text{ then } S[[\mathbf{S}]]conf \text{ else } conf$$

$$S[[\mathbf{if} \mathbf{C} \text{ then } \mathbf{S}_1 \text{ else } \mathbf{S}_2 \text{ end if}]]conf = \text{if } C[[\mathbf{C}]](conf \downarrow 1) \text{ then } S[[\mathbf{S}_1]]conf \text{ else } S[[\mathbf{S}_2]]conf$$

$$S[[\mathbf{input} \mathbf{V}]]conf = \text{if } file_1 = nil \text{ then } \top \text{ else } \langle store [hd(file_1) / \mathbf{V}], tl(file_1), file_2 \rangle$$

$$\text{where } store = conf \downarrow 1, file_1 = conf \downarrow 2, file_2 = conf \downarrow 3$$

$$S[[\mathbf{output} \mathbf{V}]]conf = \text{if } store [[\mathbf{V}]] = \perp \text{ then } \top \text{ else } \langle store, file_1, append(file_2, list(store [[\mathbf{V}]]) \rangle$$

$$\text{where } store = conf \downarrow 1, file_1 = conf \downarrow 2, file_2 = conf \downarrow 3$$

$$C[[\mathbf{E}_1 < \mathbf{E}_2]]store = \text{if } E[[\mathbf{E}_1]]store < E[[\mathbf{E}_2]]store \text{ then } true \text{ else } false$$

$$C[[\mathbf{E}_1 \leq \mathbf{E}_2]]store = \text{if } E[[\mathbf{E}_1]]store \leq E[[\mathbf{E}_2]]store \text{ then } true \text{ else } false$$

$$C[[\mathbf{E}_1 = \mathbf{E}_2]]store = \text{if } E[[\mathbf{E}_1]]store = E[[\mathbf{E}_2]]store \text{ then } true \text{ else } false$$

$$C[[\mathbf{E}_1 \neq \mathbf{E}_2]]store = \text{if } E[[\mathbf{E}_1]]store \neq E[[\mathbf{E}_2]]store \text{ then } true \text{ else } false$$

$$C[[\mathbf{E}_1 > \mathbf{E}_2]]store = \text{if } E[[\mathbf{E}_1]]store > E[[\mathbf{E}_2]]store \text{ then } true \text{ else } false$$

$$C[[\mathbf{E}_1 \geq \mathbf{E}_2]]store = \text{if } E[[\mathbf{E}_1]]store \geq E[[\mathbf{E}_2]]store \text{ then } true \text{ else } false$$

$$E[[\mathbf{E}_1 + \mathbf{E}_2]]store = E[[\mathbf{E}_1]]store + E[[\mathbf{E}_2]]store$$

$$E[[\mathbf{E}_1 - \mathbf{E}_2]]store = E[[\mathbf{E}_1]]store - E[[\mathbf{E}_2]]store$$

$$E[[\mathbf{E}_1 * \mathbf{E}_2]]store = E[[\mathbf{E}_1]]store \times E[[\mathbf{E}_2]]store$$

$$E[[\mathbf{I}]]store = N [[\mathbf{I}]]$$

$$E[[\mathbf{V}]]store = \text{if } store [[\mathbf{V}]] = \perp \text{ then } \top \text{ else } store [[\mathbf{V}]]$$